Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 6, Issue 3; April-June, 2019, pp. 176-181

http://www.krishisanskriti.org/Publication.html

Trajectory Tracking of Robotic Manipulator using Terminal Sliding Mode Control

Monisha Pathak[1], Arobindra Saikia[2] and Dr. Mrinal Buragohain[3]

[1]Department of Instrumentation Engineering, Jorhat Engineering College. Jorhat, Assam, India 785007
[2]Department of Instrumentation Engineering, Jorhat Engineering College. Jorhat, Assam, India 785007
[3]Department of Electrical Engineering, Jorhat Engineering College. Jorhat, Assam, India 785007
E-mail: [1]mamup@yahoo.co.in, [2]arvind_saikia@yahoo.com, [3]mrinalburagohain@gmail.com

Abstract—*In this paper, the robotic manipulator trajectory tracking control with terminal sliding mode control (TSMC) is investigated. The controller ensures continuous finite-time control in presence of uncertainties. Design of terminal sliding mode controller for trajectory tracking control of 3dof robotic manipulator in presence of both matched and mismatched uncertainties is presented in this work. The controller is robust against bounded uncertainties and external disturbances and guaranties finite time convergence of error ensuring satisfactory stabilization as well as tracking performances.*

Index Terms: *Robotic manipulator; mismatched uncertainties; terminal sliding mode control; finite time convergence.*

1. INTRODUCTION

Robotic manipulators are highly nonlinear multiple input multiple output (MIMO) systems with strongly coupled joints. This makes the precise trajectory tracking control of robotic manipulators a very challenging task. Degree of complexity increases with higher degree of freedom(dof) manipulators. Sliding mode control(SMC)[2,7,8,9] is one of the most appropriate approach for control of robotic manipulators. It has attracted signifcant amount of interest due to its fast global convergence, simplicity of implementation, order reduction, high robustness to external disturbances and insensitivity to model errors and system parameter variations. Control of robotic manipulators using sliding mode control has a rather long history. Numerous variations have been proposed in the literature[1,3,4,5,6,10]. In conventional sliding mode switching manifolds are usually linear hyper planes which guarantee asymptotic stability[2,7,8,9]. However for faster error convergence, the sliding mode controller parameters should be chosen such that the poles of the sliding mode dynamics are far from the origin on the left half of the s-plane. But this will cause increase of gain of the controller which may cause severe chattering on the sliding motion and thus deteriorates the system performance. To solve this problem of global asymptotic stability, terminal sliding mode control (TSMC) scheme has been developed [3,4,5,6] to achieve finite time stabilization. The TSM Controller was originated from the concept, terminal attractors [4]. The TSMC was first used in [3] for finite time sliding mode control design for robotic manipulators. It was then extended to different control problems of SISO and MIMO systems including robotics [3,5,6,10,11,12,15-21]. But all these literatures limited to trajectory tracking control of 2dof manipulators only. This work illustrates tracking control of 3dof robotic manipulators using TSMC in presence of both matched and mismatched uncertainties. The controller performance is satisfactory which ensures faster and higher-precision tracking performance in presence of uncertainties and guaranties finite time convergence of error. The paper is organized as follows. In Section II the problem is formulated. The Terminal sliding mode controller is designed in Section III. The stability analysis of the controlled system is provided in Section IV. Simulation results for trajectory tracking control of a 3 degrees of freedom serial robotic manipulator are shown in Section V. Conclusion is drawn in Section VI.

2. PROBLEM FORMULATION

The dynamics of an n-link rigid robotic manipulator can be expressed by second order nonlinear vector differential equations defined in the joint space of the manipulator (Spong and Vidyasagar,1989 [13]) as given below

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where $M(q) \in R^{n\times n}$ is the mass matrix; $C(q,\dot{q}) \in R^{n\times n}$ is the vector including centrifugal and Coriolis forces; $G(q) \in R^n$ is the gravity force vector; $\tau \in R^n$ denotes the joint torque vector $q \in R^n$ is the joint angle vector and $\dot{q}$ and $\ddot{q}$ are the angular velocity and the angular acceleration of the joints respectively.

The robotic manipulator system (1) has the following properties.

Property I :

The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in R^n$ i.e, $M(q) = M(q)^T$ and $M(q) > 0$ and it is upper and lower bounded i.e.,

$$\mu_1 I \le M(q) \le \mu_2 I \quad (2)$$
$$m_1 \le \|M(q)\| \le m_2$$

where μ_1 and μ_2 are scalars that may be computed for any given arm. Likewise, the inverse of the inertia matrix is bounded since

$$\frac{1}{\mu_2} I \le M(q)^{-1} \le \frac{1}{\mu_1} I \quad (3)$$

Property II :

Matrix $\dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix. i.e.,

$$x^T \left[\frac{1}{2}\dot{M}(q) - 2C(q,\dot{q}) \right] x = 0, \forall x \ne 0 \quad (4)$$

Assumption 1: All the joints of the robotic manipulator are revolute. This assumption makes Property 1 valid.

Assumption 2: The reference trajectory defined as $q_d \in R^n$ and its time derivatives $\dot{q}_d$ and $\ddot{q}_d$ are continuous and bounded.

3. CONTINUOUS TERMINAL SLIDING MODE CONTROLLER DESIGN

The Continuous TSMC as proposed by S.Yu, X.Yu, B.S. and Z. man [2005] [6] is as explained below.

The TSM and Fast TSM is given by the following first order nonlinear differential equations

$$s = \dot{x} + \beta |x|^{\gamma} sign(x) = 0$$
$$s = \dot{x} + \alpha x + \beta |x|^{\gamma} sign(x) = 0 \quad (5)$$

respectively, where $x \in R, \alpha, \beta > 0, 0 < \gamma < 1$.

The equilibrium point $x = 0$ of the above equation (5) is globally finite-time stable, i.e., for any given initial condition $x(0) = x_o$, the system state converges to $x = 0$ in finite time T as given below for TSM and FTSM

$$T = \frac{1}{\beta(1-\gamma)} |x_o|^{1-\gamma}$$
$$T = \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha |x_o|^{1-\gamma} + \beta}{\beta} \quad (6)$$

respectively and stays there forever, i.e. $x = 0$ for $t > T$.

An extended Lyapunov description of finite time stability can be given with the form of fast TSM as

$$\dot{V}(x) + \alpha V(x) + \beta V^{\gamma}(x) \le 0, 0 < \gamma < 1 \quad (7)$$

and the settling time can be given by

$$T \le \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha V^{1-\gamma}(x_o) + \beta}{\beta} \quad (8)$$

It is clear that above equations (7) and (8) mean exponential stability as well as faster finite-time stability.

And the NTSM can be expressed as

$$s = x + \beta |\dot{x}|^{\gamma} sign(\dot{x}) = 0 \quad (9)$$

where $\beta > 0$, and $1 < \gamma < 2$. It is continuous and differentiable although the absolute value and signum operators are involved. Its first derivative can be expressed as

$$\dot{s} = \dot{x} + \beta\gamma |\dot{x}|^{\gamma-1} \ddot{x} \quad (10)$$

Let $q_d \in R^n$ be a twice differentiable desired trajectory, and define the tracking error as $e = q - q_d$. The control objective is to find a feedback control τ such that the manipulator output q tracks the desired trajectory q_d in finite time e.g. to make the tracking error zero.

The following notions are introduced for simplicity of expression in developing TSM (Haimo 1986)[14]:

$$y^{\gamma} = [y_1^{\gamma_1}, \ldots\ldots, y_n^{\gamma_n}]^T$$
$$|y|^{\gamma} = [|y_1|^{\gamma_1}, \ldots\ldots, |y_n|^{\gamma_n}]^T$$
$$sig(y)^{\gamma} = [|y_1|^{\gamma_1} sign(y_1), \ldots\ldots, |y_n|^{\gamma_n} sign(y_n)]^T$$

Hence, the Nonsingular TSM as from (9) can be defined as

$$s = e + \beta sig(\dot{e})^{\gamma} = 0 \quad (11)$$

Where $s = [s_1, \ldots, s_n]^T \in \Re^n$, $\beta = diag(\beta_1, \ldots\ldots, \beta_n)$ and $1 < \gamma_1, \ldots\ldots, \gamma_n < 2$

$$\dot{s} = \dot{e} + \beta diag(\gamma_1 |\dot{e}_1|^{\gamma_1 - 1}, \ldots, \gamma_n |\dot{e}_n|^{\gamma_n - 1})$$
$$(M(q)^{-1}(\tau - C(q,\dot{q})\dot{q} - G(q)) - \ddot{q}_d) \quad (12)$$

The conventional TSM control can be designed as a discontinuous control law according to a discontinuous reaching law such as

$\dot{s} = -ksign(s)$ (13)

where $k = diag(k_1,........,k_n)$, $i = 1,........,n$ and $sign(s) = [sign(s_1),......,sign(s_n)]^T$ .A discontinuous TSM control can be designed as

$$\tau = C(q,\dot{q})\dot{q} + G(q) \\ -M(q)(ksign(s) - \ddot{q}_d + \beta^{-1}\gamma^{-1}|\dot{e}|^{2-\gamma}) \quad (14)$$

Retaining the property of finite-time reaching of TSM but eliminating discontinuities,[6] has proposed a kind of continuous fast-TSM-type reaching condition as

$\dot{s} = -k_1 s - k_2 sig(s)^\rho$ (15)

Where $k_1 = diag(k_{11},........,k_{1n})$, $k_2 = diag(k_{21},........,k_{2n})$, $k_{1i}, k_{2i} > 0, 0 < \rho = \rho_1 = ... = \rho_n < 1$. The inverse dynamics controller is designed as

$$\tau = C(q,\dot{q})\dot{q} + G(q) \\ -M(q)(k_1 s + k_2 sig(s)^\rho - \ddot{q}_d + \beta^{-1}\gamma^{-1}|\dot{e}|^{2-\gamma}) \quad (16)$$

This control law is continuous and therefore is chatteringfree. It does not involve any negative fractional power, hence it is also singularity-free.

4. STABILITY ANALYSIS

The Lyapunov function is considered as $V = \frac{1}{2}s^T s$. By differentiating V with respect to time,

$$\begin{aligned}\dot{V} &= s^T\dot{s} \\ &= s^T[\dot{e} + \beta\gamma diag(|\dot{e}|^{\gamma-1})\ddot{e}] \\ &= s^T[\dot{e} + \beta\gamma diag(|\dot{e}|^{\gamma-1}) \\ &\quad (M(q)^{-1}(\tau - C(q,\dot{q})\dot{q} - G(q)) - \ddot{q}_d)] \quad (17) \\ &= s^T[-\beta\gamma diag(|\dot{e}|^{\gamma-1})k_1 s - \beta\gamma diag(|\dot{e}|^{\gamma-1})k_2 sig(s)^\rho] \\ &= -s^T K_x s - s^T K_y sig(s)^\rho \\ &< 0\end{aligned}$$

Where $K_x = \beta\gamma diag(|\dot{e}|^{\gamma-1})k_1 \in R^{n\times n}$ and $K_y = \beta\gamma diag(|\dot{e}|^{\gamma-1})k_2 \in R^{n\times n}$ are positive definite diagonal matrices. The $\dot{V} < 0$ implies the stability in Lypunov sence.

5. SIMULATION RESULTS

A planar, 3dof manipulator with revolute joints, taken from [10], is used here to demonstrate the given control approach. The manipulator and the associated variables are depicted in Figure 1 .

The model of this robot is simulated by using MATLAB Simulink platform with fixed step size of 0.001.

The robot model is defined by the following equation (Neila Mezghani Ben Romdhane and Tarak Damak 2015)[10]

$$\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}\begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} + \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} + \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \quad (19)$$

$$\begin{aligned}M_{11} &= 2b_1\cos(q_2) + 2b_2\cos(q_2+q_3) + 2b_3\cos(q_3) + a_1 \\ M_{12} &= b_1\cos(q_2) + b_2\cos(q_2+q_3) + 2b_3\cos(q_3) + a_2 \\ M_{22} &= 2b_3\cos(q_3) + a_2 \\ M_{13} &= b_2\cos(q_2+q_3) + b_3\cos(q_3) + a_3 \\ M_{23} &= a_3 + b_3\cos(q_3) \\ M_{33} &= a_3\end{aligned}$$

$$\begin{aligned}a_1 &= J_1 + m_1L_{c1}^2 + J_2 + m_2(L_1^2 + L_{c2}^2) + J_3 + m_3(L_1^2 + L_2^2 + L_{c3}^2) \\ a_2 &= J_2 + m_2L_{c2}^2 + m_3(L_2^2 + L_{c3}^2) \\ a_3 &= J_3 + m_3L_{c3}^2\end{aligned}$$

$$\begin{aligned}C_1 &= -b_1\dot{q}_2(2\dot{q}_1 + \dot{q}_2)\sin(q_2) - b_2(2\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \\ &(\dot{q}_2 + \dot{q}_3)\sin(q_2+q_3) - b_3\dot{q}_3(2\dot{q}_1 + \dot{q}_2 + \dot{q}_3)\sin(q_3) \\ C_{2=} &- b_1\dot{q}_1^2\sin(q_2) - b_2\dot{q}_1^2\sin(q_2+q_3) - b_2(2\dot{q}_1 + \dot{q}_2 + \dot{q}_3) + \sin(q_3) \\ C_3 &= -b_2\dot{q}_1^2\sin(q_2+q_3) - b_3(\dot{q}_1 + \dot{q}_2)^2\sin(q_3)\end{aligned}$$

$$\begin{aligned}b_1 &= m_2L_1L_{c2} + m_3L_1L_2 \\ b_2 &= m_3L_1L_{c3} \\ b_3 &= m_3L_2L_{c3}\end{aligned}$$

$$\begin{aligned}G_1 &= k_1\cos(q_1) + k_2\cos(q_1+q_2) + k_3\cos(q_1+q_2+q_3) \\ G_2 &= k_1\cos(q_1+q_2) + k_3\cos(q_1+q_2+q_3) \\ G_3 &= k_3\cos(q_1+q_2+q_3)\end{aligned}$$

$$\begin{aligned}k_1 &= (m_1L_{c1} + m_2L_1 + m_3L_1)g \\ k_2 &= (m_2L_{c2} + m_3L_2)g \\ k_3 &= m_3L_{c3}g\end{aligned}$$

Here $q(t) = [q_1(t), q_2(t), q_3(t)]^T$ is the angular position vector where $q_1(t)$, $q_2(t)$ and $q_3(t)$ are the angular positions of joints 1,2 and 3. $M(q)$ is the inertia matrix, $C(q,\dot{q})$ is the centripetal Coriolis matrix, $G(q)$ is the gravity vector and $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the applied torque. Friction terms are ignored.

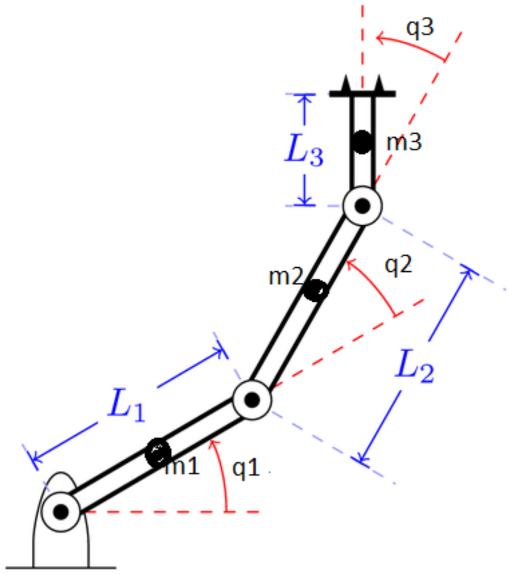


Fig. 1: Three-link manipulator with link lengths L1 and L2 , L3 and link masses m1 and m2 and m3 [10].

TABLE I: Physical parameters of the three-link robotic manipulator [10]

Symbol	Definition	Value
L1	Length of the first link	0.5m
L2	Length of the second link	0.5m
Lc1	Distance to the link COM	0.25m
Lc2	Distance to the link COM	0.35m
Lc3	Distance to the link COM	0.15m
J1	Moment of inertia of the D.C. motor	$0.12 kgm^2$
J2	Moment of inertia of the D.C. motor	$0.25 kgm^2$
J3	Moment of inertia of the D.C. motor	$0.3 kgm^2$
m1	Nominal Mass of the link 1	0.5kg
m2	Nominal Mass of the link 2	1kg
m3	Nominal Mass of the link 3	0.2kg
g	Gravitational constant	$9.81 m/s^2$

Table I lists the physical parameters of the manipulator considered in the simulation study[10].

Each joint of the robot arm is required to track the time varying reference trajectory $q_d = 0.2 + 2\sin(2t)$. Let us suppose that we have an uncertainty on masses of the

order ±10% and an uniform random noise having limits ±0.0001 is added to the measurements of position and speed of the joints. The controller performance is studied

when the robotic arm is affected by these mismatched uncertainties.

The parameters selected for the terminal sliding mode controller (16) are $\beta = diag(3,3,3)$, $\gamma = 1.5$, $k_1 = k_2 = diag(35,35,35)$ and $\rho = 0.9$.

The simulation results are shown in APPENDIX I. From the simulation results it is observed that a continuous control action is obtained which is able to keep the system on the desired trajectory despite the uncertainties present in the system. It is also observed that the tracking performance is satisfactory with small rise time. Moreover, the sliding surface and error convergence is also satisfactory. To discuss the controller performance the output performance parameter integrated absolute error (IAE) is calculated from the results and is tabulated in table II.

TABLE II: Controller performance

Controller Performance	
Joint	IAE
Joint1	0.05503
Joint2	0.05513
Joint3	0.05502

6. CONCLUSIONS

A continuous nonsingular terminal sliding mode controller is presented in this work. The proposed controller exhibits satisfactory tracking performance for the higher degrees of freedom robotic arm even in presence of mismatched uncertainties caused by parametric change and sensor noise. Simulation studies conducted on a 3 dof robotic arm shows effectiveness of the proposed controller. The controller is chattering free and singularity free.

REFERENCES

[1] J.-J. E. Slotine and W. Li, Composite adaptive control of robot manipulators , Automatica, vol. 25, no. 4, pp. 509–519, 1989.

[2] C. Edwards and S. Spurgeon, Sliding mode control: theory and applications , CRC Press, 1998.

[3] S. Venkataraman and S. Gulati, Terminal slider control of robot systems ,Journal of Intelligent and Robotic systems, vol. 7, no.1, pp.31–55, 1993.

[4] M. Zak, Terminal attractors in neural networks,Neural Networks, vol. 2, no. 4, pp. 259 – 274, 1989.

[5] Y. Feng, X. Yu, and Z. Man, Non-singular terminal sliding mode control of rigid manipulators,Automatica, vol. 38, no.12, pp. 2159– 2167, 2002.

[6] S. Yu, X. Yu, B. Shirinzadeh, and Z. Man, Continuous finite time control for robotic manipulators with terminal sliding mode, Automatica, vol. 41, no. 11, pp. 1957 – 1964,2005.

[7] V. I. Utkin, Sliding modes in control and optimization,Springer Science Business Media, 2013.

[8] V. Utkin, J. Guldner, and J. Shi, Sliding mode control in electromechanical systems,CRC press, 2009, vol. 34.

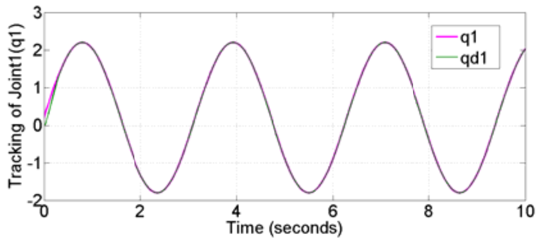
[9] V. Utkin, J. Guldner, and J. Shi, Sliding mode control in electromechanical systems, CRC press, 2009, vol. 34.

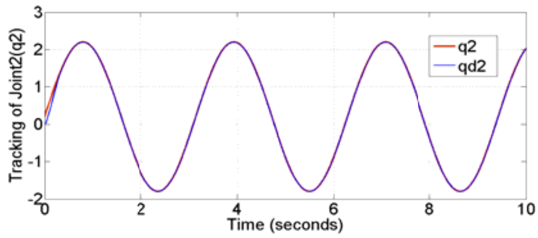
[10] Neila Mezghani Ben Romdhane,Tarak Damak,Higher Order Sliding Mode Control of Uncertain Robot Manipulators, Advances and Applications in SlidingMode Control systems,Studies in Computational Intelligence, Springer, Volume 576,pp.327–352,2015.

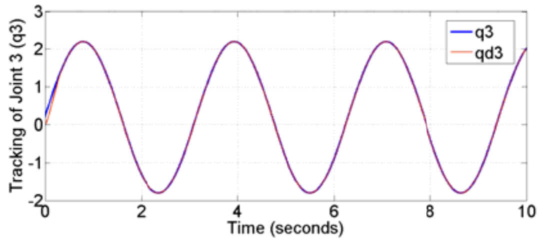
[11] V. Nekoukar and A. Erfanian,Adaptive fuzzy terminal sliding mode control for a class of uncertain nonlinear systems, Fuzzy Sets and Systems, vol. 179, no. 1, pp. 34 – 49, 2011.

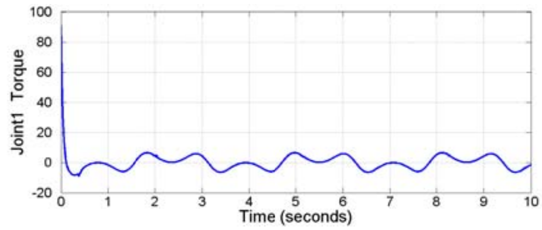
[12] D. Zhao, S. Li, and F. Gao,A new terminal sliding modecontrol for robotic manipulators,, International Journal ofControl, vol. 82, no.10,pp. 1804–1813, 2009.
[13] M. W. Spong and M. Vidyasagar,Robot dynamics and control., John Wiley Sons, 2008.
[14] V. T. Haimo,Finite time controllers, SIAM Journal on Control and Optimization,vol. 24, no. 4, pp. 760–770, 1986
[15] Mirosław Galicki,Finite-time control of robotic manipulators, Automatica,V-51,49–54,2015.
[16] Zuoshi Song, Jianqiang Yi, Dongbin Zhao, Xinchun Li,A computed torque controller for uncertain robotic manipulator systems: Fuzzy approachFuzzy Sets and Systems 154 (2005) 208–226, Elsevier.
[17] Chuan-Kai Lin,Nonsingular Terminal Sliding Mode Control of Robot Manipulators Using Fuzzy Wavelet Networks IEEE Transactions on Fuzzy Systems (Volume:14 , Issue: 6),849 - 859,2006
[18] Mehmet Onder Efe, Fractional Fuzzy Adaptive Sliding-Mode Control of a 2-DOF Direct-Drive Robot ArmIEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics),(Volume:38,Issue: 6),1561 - 1570,2008
[19] Shafiqul Islam,Xiaoping P. Liu,Robust Sliding Mode Control for Robot ManipulatorsIEEE Transactions on Industrial Electronics(Volume:58,Issue: 6),2444 - 2453,2010
[20] Daoxiang Gao,Dingyu Xue,Terminal Sliding Mode Adaptive Control for Robotic Manipulators,2006 6th World Congress on Intelligent Control and Automation (Volume:2),8853 - 8857
[21] Tingzhe Jia ,Gewen Kang,An RBF Neural Network-Based Nonsingular Terminal Sliding Mode Controller for Robot Manipulators,Intelligent Control and Information Processing (ICICIP),2012

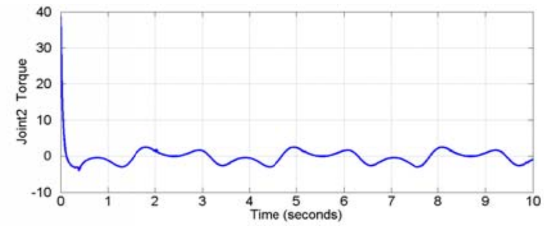
APPENDIX I

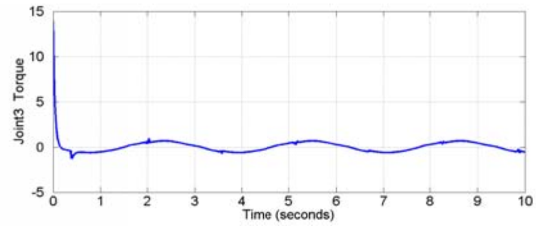


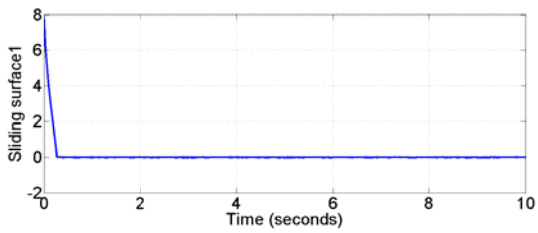


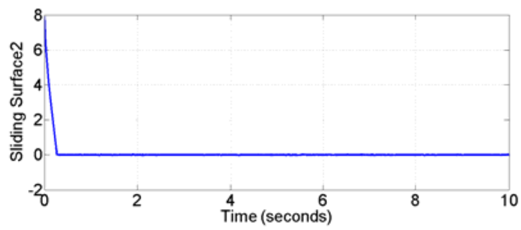


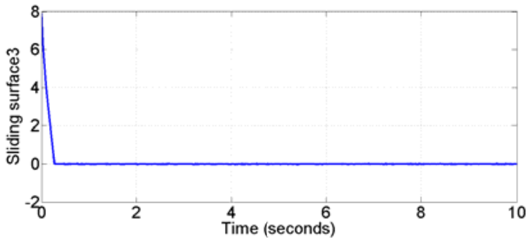


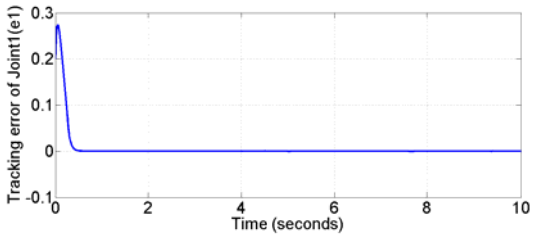


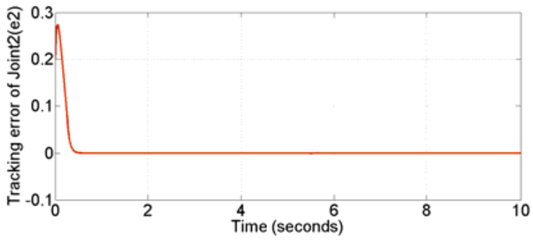


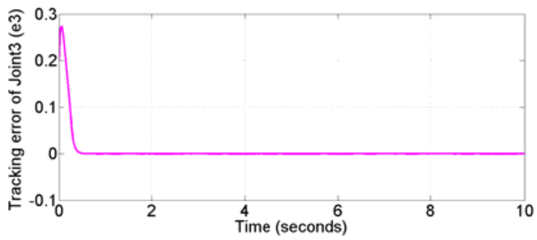





Sliding surface3
Time (seconds)


Tracking error of Joint1(e1)
Time (seconds)


Tracking error of Joint2(e2)
Time (seconds)


Tracking error of Joint3 (e3)
Time (seconds)